Single Pure – Factor Theorem Cubic Solving

1. The equation $f(x) = x^3 - 4x^2 + x + 6 = 0$ has three integer roots.
   (a) List the values of a for which it is sensible to check whether $f(a) = 0$ and check each of them.
   (b) Solve $f(x) = 0$.

2. By considering f(various sensible values), fully factorise the following:
   (a) $2x^3 + 5x^2 - x - 6$. $(2x + 3)(x - 1)(x + 2)$
   (b) $3x^3 - 2x^2 - 7x - 2$. $(3x + 1)(x + 1)(x - 2)$
   (c) $2x^4 - 9x^3 + x^2 + 12x$. $x(x - 4)(2x - 3)(x + 1)$
   (d) $6x^4 - 7x^3 - 12x^2 + 3x + 2$. $(x - 2)(x + 1)(2x - 1)(3x + 1)$

3. Solve the following equations:
   (a) $x^3 + 3x^2 = 10x$. $x = 0$ or $x = 2$ or $x = -5$
   (b) $x^3 + 12 = 2x^2 + 11x$. $x = 1$ or $x = -3$ or $x = 4$
   (c) $x^3 + 31x = 10x^2 + 30$. $x = 5$ or $x = 2$ or $x = 3$
   (d) $2 = 2x^3 + 5x^2 + x$. $x = \frac{1}{2}$ or $x = -1$ or $x = -2$
   (e) $3x^3 + 11x^2 + 8x = 4$. $x = \frac{1}{3}$ or $x = -2$ (repeated)
   (f) $x^3 + x^2 = 10 + x$. $x = 2$ (only)
   (g) $x^3 + 2x^2 - 3 = 2x$. $x = -1$ or $x = \frac{-1 \pm \sqrt{13}}{2}$
   (h) $2x^3 + x^2 + 1 = 3x$. $x = \frac{1}{2}$ or $x = \frac{-1 \pm \sqrt{5}}{2}$
   (i) $x^4 + 17x^2 + 2x = 8x^3 + 24$. $x = -1$ or $x = 2$ or $x = 3$ or $x = 4$
   (j) $y^4 + 3y^3 = 13y^2 + 51y + 36$. $y = -1$ or $y = -3$ (repeated) or $y = 4$
   (k) $x^4 + x^3 + x + 3 = 6x^2$. $x = -3$ or $x = 1$ or $x = \frac{1 \pm \sqrt{5}}{2}$
   (l) $7x^3 + 22x = 2x^4 + 15x^2 + 8$. $x = 2$ or $x = \frac{1}{2}$

4. Given that $(x - 4)$ is a factor of $2x^3 - 5x^2 - 14x + a$, find a. $a = 8$

5. Given that $(x + 3)$ is a factor of $3x^3 + 9x^2 + bx + 6$, find b. $b = 2$

6. Given that $(2x - 1)$ is a factor of $2x^3 + cx^2 + 8x - 2$, find c. $c = -9$

7. Given that $(2x - 1)$ is a factor of $4x^4 + ax^3 + x^2 + 9x - 3$, find a. $a = -16$

8. Given that $(x + 3)$ and $(x - 2)$ are factors of $2x^3 + dx^2 + ex - 6$, find d and e. $d = 3, e = -11$

9. Given that $(x - 2)$ and $(x + 3)$ are factors of $2x^3 + x^2 + ax + b$, find a and b. $a = -13, b = 6$

10. Given that $(x + 1)$ and $(x - 5)$ are factors of $x^3 + ax^2 + bx - 20$, find a and b. $a = 0, b = -21$

11. Given that $(x - 6)$ and $(x + 2)$ are factors of $x^3 + ax^2 + bx - 24$, find a and b. $a = -2, b = -20$

12. Given that $(x - 1)$ and $(x - 2)$ are factors of $x^4 + ax^3 + bx^2 - 27x + 10$, find a and b. $a = -9, b = 25$

13. Given that $(x + 1)$ and $(x - 4)$ are factors of $x^4 + ax^3 - 13x^2 + bx - 36$, find a and b. $a = 3, b = -51$

14. Given that $(x+1)$, $(x-2)$, and $(x-3)$ are factors of $x^4 + ax^3 + bx^2 + cx + 18$, find a, b and c.

$a = -1, b = -11, c = 9$

15. Given that $(x+1)$, $(x+3)$, and $(x-4)$ are factors of $x^4 + ax^3 + bx^2 + cx - 60$, find a, b and c.

$a = 5, b = -13, c = -77$